PHYSICAL STRUCTURES.
FORMING PHYSICAL FIELDS AND MANIFOLDS
(Properties of skew-symmetric differential forms)

L.I. Petrova

Moscow State University, Russia, e-mail: ptr@cs.msu.su

It is shown that physical fields are formed by physical structures, which in their properties are differential-geometrical structures.

These results have been obtained due to using the mathematical apparatus of skew-symmetric differential forms. This apparatus discloses the controlling role of the conservation laws in evolutionary processes, which proceed in material media and lead to origination of physical structures and forming physical fields and manifolds.

1 Physical structures

The closure conditions of the inexact exterior differential form and dual form (the equality to zero of differentials of these forms) can be treated as a definition of some differential-geometrical structure. In this section it will be shown that as the physical structures, which form physical fields, it serve those, which in their properties are such differential-geometrical structures.

The properties of such differential-geometrical structures, and correspondingly physical structures, are based on the properties of closed exterior differential forms.

Below the properties of closed exterior differential forms are briefly described. (In more detail about skew-symmetric differential forms one can read in [1,2]. With the theory of exterior differential forms one can become familiar from the works [3-7]).

Closed exterior differential forms

The exterior differential form of degree p (p-form on the differentiable manifold) is called a closed one if its differential equals zero:

$$d\theta^p = 0 \tag{1}$$

From condition (1) one can see that the closed form is a conservative quantity. This means that such a form can correspond to the conservation law, namely, to some conservative physical quantity.

If the form is closed on pseudostructure only (i.e. it is the closed *inexact* differential form), the closure condition is written as

$$d_\pi \theta^p = 0 \tag{2}$$

And the pseudostructure π is defined from the condition

$$d_\pi{}^*\theta^p = 0 \qquad (3)$$

where ${}^*\theta^p$ is the dual form.

{Cohomology, sections of cotangent bundles, the eikonal surfaces, the characteristical and potential surfaces, and so on can be regarded as examples of pseudostructures. For the properties of dual forms see [7].}

From conditions (2) and (3) one can see that the exterior differential form closed on pseudostructure (a closed inexact form) is a conservative object, namely, this quantity conserves on pseudostructure. This can also correspond to some conservation law, i.e. to conservative object. Such conservation laws that state the existence of conservative physical quantities or objects can be named exact ones.

The pseudostructure and the closed exterior form defined on the pseudostructure form a differential-geometrical structure. (It can be noted that this structure is the example of the differential-geometrical G-Structures). It is evident that just such structures, which correspond to the exact conservation law, are physical structures, from which physical fields are formed. {The physical fields [6] are a special form of the substance, they are carriers of various interactions such as electromagnetic, gravitational, wave, nuclear and other kinds of interactions.}

The problem of how these structures arise and how physical fields are formed will be discussed below.

Thus, the closure conditions for the exterior differential form ($d_\pi\,\theta^p = 0$) and the dual form ($d_\pi\,{}^*\theta^p = 0$) are mathematical expressions of the exact conservation law and they define the physical structure.

The mathematical expression for the exact conservation law and its connection with physical fields can be schematically written in the following way

$$\begin{cases} d_\pi\theta^p = 0 \\ d_\pi{}^*\theta^p = 0 \end{cases} \mapsto \begin{cases} \theta^p \\ {}^*\theta^p \end{cases} \text{---} \quad \text{physical structures} \quad \mapsto \quad \text{physical fields}$$

Since the relations for exact conservation laws and for relevant physical structures (that form physical fields) are expressed in terms of the closed exterior and dual forms, it is evident the field theories (which describe physical fields) are based on the mathematical apparatus of the closed exterior differential and dual forms.

One can express the field theory operators in terms of following operators of exterior differential forms: d (exterior differential), δ (the operator of transforming the form of degree $p+1$ into the form of degree p), δ' (the operator of cotangent transforms), Δ (that of the transformation $d\delta - \delta d$), Δ' (the operator of the transformation $d\delta' - \delta'd$). In terms of these operators, which act onto exterior forms, one can write down the operators by Green, d'Alembert, Laplace and the operator of canonical transform [7].

The equations, that are equations of the existing field theories, are those obtained on the basis of the properties of the exterior differential form theory.

It can be shown that to the quantum mechanical equations (to the equations by Shrődinger, Heisengerg and Dirac) there correspond the closed exterior forms of zero degree or the relevant dual forms. The closed exterior form of zero degree corresponds to the Schrődinger equation, the close dual form corresponds to the Heisenberg equation. It can be pointed out that, whereas the equations by Schrődinger and Heisenberg describe a behavior of the potential obtained from the zero degree closed form, Dirac's *bra-* and *cket-* vectors constitute a zero degree closed exterior form itself as the result of conjugacy (vanishing the scalar product).

The Hamilton formalism is based on the properties of closed exterior and dual forms of the first degree. The closed exterior differential form $ds = -Hdt + p_jdq_j$ (the Poincare invariant) corresponds to the field equation.

The properties of closed exterior and dual forms of the second degree lie at the basis of the electromagnetic field equations. The Maxwell equations may be written as $d\theta^2 = 0$, $d^*\theta^2 = 0$ [7], where $\theta^2 = \frac{1}{2}F_{\mu\nu}dx^\mu dx^\nu$ (here $F_{\mu\nu}$ is the strength tensor).

Closed exterior and dual forms of the third degree correspond to the gravitational field.

From the above stated one can see that to each type of physical fields there corresponds a closed exterior form of appropriate degree.

The connection of the physical structures with exterior forms allows to understand the properties and specific features of the physical structures.

2 The properties of exterior differential forms and physical structures

Basic properties of exterior differential forms are connected with the fact that any closed form is a differential.

The exact form is, by definition, a differential

$$\theta^p = d\theta^{p-1} \tag{4}$$

In this case the differential is total. The closed inexact form is a differential too. The closed inexact form is an interior (on pseudostructure) differential, that is

$$\theta^p_\pi = d_\pi\theta^{p-1} \tag{5}$$

And so, any closed form is a differential of the form of a lower degree: the total one $\theta^p = d\theta^{p-1}$ if the form is exact, or the interior one $\theta^p = d_\pi\theta^{p-1}$ on pseudostructure if the form is inexact. (This may have the physical meaning: the form of lower degree can correspond to the potential, and the closed form by itself can correspond to the potential force.)

Invariant properties of closed exterior differential forms and physical structures. Nondegenerate transformations

Since the closed form is a differential, then it is evident that the closed form proves to be invariant under all transformations that conserve a differential.

The examples of such nondegenerate transformations are unitary, tangent, canonical, and gradient transformations.

To the nondegenerate transformations there are assigned closed forms of given degree. To the unitary transformations it is assigned (0-form), to the tangent and canonical transformations it is assigned (1-form), to the gradient transformations it is assigned (2-form) and so on. It should be noted that these transformations are *gauge transformations* for spinor, scalar, vector, tensor (3-form) fields. Hence one can see that the physical structure relate to the gauge type of the differential-geometrical G-Structure. They remain to be invariant under all transformations that conserve the differential.

It is well known that these are transformations typical for existing field theories. The equations of existing field theories remain invariant under such transformations.

The closure of exterior differential forms, and hence their invariance, results from the conjugacy of elements of exterior or dual forms.

From the definition of exterior differential form one can see that exterior differential forms have complex structure. Specific features of the exterior form structure are homogeneity with respect to the basis, skew-symmetry, integrating terms each including two objects of different nature (the algebraic nature for form coefficients, and the geometric nature for base components). Besides, the exterior form depends on the space dimension and on the manifold topology. The closure property of the exterior form means that any objects, namely, elements of the exterior form, components of elements, elements of the form differential, exterior and dual forms and others, turn out to be conjugated. (In the author's work [8] some types of conjugacy of exterior differential forms have been considered). A variety of objects of conjugacy leads to the fact that the closed forms can describe a great number of various physical structures.

Since the conjugacy is a certain connection between two operators or mathematical objects, it is evident that, to express a conjugacy mathematically, it can be used relations. Just such relations constitute the basis of mathematical apparatus of the exterior differential forms. This is an identical relation. Identical relations of exterior differential forms also disclose the properties of physical structures.

Identical relations of exterior differential forms

Identical relations of exterior differential forms reflect the closure conditions of differential forms, namely, vanishing the form differential (see formulas (1), (2), (3)) and hence the conditions connecting the forms of consequent degrees (see

formulas (4), (5)). Since the closure conditions of differential forms and dual forms specify the differential-geometrical structures and physical structures, identical relations for exterior differential forms specify the physical structures.

The identical relations are a mathematical expression of the invariance and covariance. And this lies at the basis of existing field theories.

Examples of such relations are canonical relations in the Schrődinger equations, gauge invariance in electromagnetic theory, commutator relations in the Heisenberg theory, symmetric connectednesses, identity relations by Bianchi in the Einstein theory, cotangent bundles in the Yang-Mills theory, the covariance conditions in the tensor methods, the characteristic relations (integrability conditions) in equations of mathematical physics, etc. (In more detail about identical (and nonidentical) relations it is outlined in the author's work [8]).

The identical relations express the fact that each closed exterior form is a differential of some exterior form (with a degree less by one). In general form such an identical relation can be written as

$$d_\pi \phi = \theta_\pi^p \tag{6}$$

In this relation the form in the right-hand side has to be a *closed* one. (As it will be shown below, the identical relations are satisfied only on pseudostructures).

In identical relation (6) in one side it stands the closed form and in other side does a differential of some differential form of the less by one degree, which is a closed form as well.

The identical relations of another type are the analog of relation (6) obtained by differentiating or integrating this relation.

3 Mechanism of origination of physical structures

It has been shown that the skew-symmetric closed exterior differential forms allow to describe the properties and specific features of the physical structures.

In this section it will be shown that the skew-symmetric differential forms describe also the process of origination of physical structures. However, to do this, one must use skew-symmetric differential forms, which, in contrast to exterior (skew-symmetric) differential forms, possess the evolutionary properties, and for this reason they were named evolutionary differential forms.

A peculiarity of the evolutionary differential forms consists in the fact that they generate exterior differential forms, which correspond to physical structures. This elucidates the process of origination of physical structures.

Specific features of the evolutionary differential forms

A radical distinction between the evolutionary forms and the exterior ones consists in the fact that the exterior differential forms are defined on manifolds with

closed metric forms, whereas the evolutionary differential forms are defined on manifolds with *unclosed metric forms*.

The closed metric forms define the manifold structure, and the commutators of metric forms define the manifold differential characteristics that specify the manifold deformation: bending, torsion, rotation, twist.

The theory of exterior differential forms was developed for both differentiable manifolds and manifolds with structures of any types as well. It is evident that the manifolds, which are metric ones or possess the structure, have closed metric forms. All have one common property, namely, locally they admit one-to-one mapping into Euclidean subspaces and into other manifolds or submanifolds of the same dimension [5].

When describing any processes in terms of differential equations, one has to deal with manifolds that do not allow one-to-one mapping described above. Such manifolds are, for example, manifolds formed by trajectories of elements of the system described by differential equations. The manifolds that can be called accompanying manifolds are variable deforming manifolds.

If the manifolds are deforming manifolds, this means that their metric form commutators are nonzero. That is, the metric forms of such manifolds turn out to be unclosed. The accompanying manifolds and manifolds occurring to be deforming ones are examples of such manifolds.

The skew-symmetric evolutionary differential forms, whose basis are deforming manifolds, are defined on manifolds with unclosed metric forms.

The evolutionary properties of the evolutionary skew-symmetric differential forms are just connected with properties of the metric form commutators.

The evolutionary differential form of degree p (p-form) can be also written as an exterior differential form [1].

But the evolutionary form differential cannot be written similarly to that presented for exterior differential forms. In the evolutionary form differential there appears an additional term connected with the fact that the basis of the form changes [1].

For example, we again inspect the first-degree form $\omega = a_\alpha dx^\alpha$. [From here on the symbol $\sum$ will be omitted and it will be implied that a summation over double indices is performed. Besides, the symbol of exterior multiplication will be also omitted for the sake of presentation convenience].

The differential of this form can be written as $d\omega = K_{\alpha\beta} dx^\alpha dx^\beta$, where $K_{\alpha\beta} = a_{\beta;\alpha} - a_{\alpha;\beta}$ are components of the commutator of the form ω, and $a_{\beta;\alpha}$, $a_{\alpha;\beta}$ are the covariant derivatives. If we express the covariant derivatives in terms of the connectedness (if it is possible), they can be written as $a_{\beta;\alpha} = \partial a_\beta/\partial x^\alpha + \Gamma^\sigma_{\beta\alpha} a_\sigma$, where the first term results from differentiating the form coefficients, and the second term results from differentiating the basis. (In Euclidean space covariant derivatives coincide with ordinary ones since in this case derivatives of the basis vanish). If we substitute the expressions for covariant derivatives into the formula for the commutator components, we obtain the

following expression for the commutator components of the form ω:

$$K_{\alpha\beta} = \left(\frac{\partial a_\beta}{\partial x^\alpha} - \frac{\partial a_\alpha}{\partial x^\beta} \right) + (\Gamma^\sigma_{\beta\alpha} - \Gamma^\sigma_{\alpha\beta})a_\sigma \tag{7}$$

Here the expressions $(\Gamma^\sigma_{\beta\alpha} - \Gamma^\sigma_{\alpha\beta})$ entered into the second term are just the components of commutator of the first-degree metric form.

The evolutionary form commutator of any degree involves the commutator of the manifold metric form of corresponding degree. The commutator of the exterior form does not contains a similar term because the commutator of metric form of manifold, on which the exterior form is defined, is equal to zero.

The commutators of evolutionary forms depend not only on the evolutionary form coefficients, but also on the characteristics of manifolds, on which this form is defined. As a result, such a dependence of the evolutionary form commutator produces the topological and evolutionary properties of both the commutator and the evolutionary form itself (this will be demonstrated below).

The evolutionary differential form commutator, in contrast to that of the exterior one, cannot be equal to zero since it includes the metric form commutator being nonzero. This means that the evolutionary form differential is nonzero. Hence, *the evolutionary differential form, in contrast to the case of the exterior form, cannot be closed.*

It was noted above that the closed exterior differential forms describe the conservation laws. It appears that the evolutionary differential forms, which are unclosed, describe the conservation laws also [9]. The difference consists in the fact that the closed exterior differential forms describe the conservation laws for physical fields (exact conservation laws that state the existence of conservative physical quantities or objects), whereas the evolutionary differential forms describe the conservation laws for material media (material systems). {The material system is a variety of elements that have internal structure and interact to one another. As examples of material systems it may be thermodynamic, gas dynamical, cosmic systems, systems of elementary particles (pointed above) and others. Any material media are such material systems. Examples of elements that constitute the material system are electrons, protons, neutrons, atoms, fluid particles, cosmic objects and others}.

The conservation laws for material systems establish the balance between the variation of a physical quantity and the corresponding external action. These conservation laws, which can be called the balance conservation laws, are the conservation laws for energy, linear momentum, angular momentum, and mass.

The properties of evolutionary differential forms, which correspond to the balance conservation laws, lie at the basis of the processes of forming physical fields and corresponding manifolds. To understand a mechanism of these processes one should look at the properties of the balance conservation laws.

Properties of the balance conservation laws

Equations of the balance conservation laws are differential (or integral) equations that describe a variation of functions corresponding to physical quantities [10-13]. From the equations, which describe the balance conservation laws, the evolutionary relation in differential forms is obtained.

Let us analyze the equations that describe the balance conservation laws for energy and linear momentum.

We introduce two frames of reference: the first is an inertial one (this frame of reference is not connected with the material system), and the second is an accompanying one (this system is connected with the manifold constructed of the trajectories of the material system elements). The energy equation in the inertial frame of reference can be reduced to the form:

$$\frac{D\psi}{Dt} = A_1 \tag{8}$$

where D/Dt is the total derivative with respect to time, ψ is the functional of the state that specifies the material system, A_1 is the quantity that depends on specific features of the system and on external energy actions onto the system. {The action functional, entropy, wave function can be regarded as examples of the functional ψ. Thus, the equation for energy presented in terms of the action functional S has a similar form: $DS/Dt = L$, where $\psi = S$, $A_1 = L$ is the Lagrange function. In mechanics of continuous media the equation for energy of ideal gas can be presented in the form [11]: $Ds/Dt = 0$, where s is entropy. In this case $\psi = s$, $A_1 = 0$. It is worth noting that the examples presented show that the action functional and entropy play the same role.}

In the accompanying frame of reference the total derivative with respect to time is transformed into the derivative along the trajectory. Equation (8) is now written in the form

$$\frac{\partial \psi}{\partial \xi^1} = A_1 \tag{9}$$

here ξ^1 is the coordinate along the trajectory.

In a similar manner, in the accompanying frame of reference the equation for linear momentum turns out to be reduced to the equation of the form

$$\frac{\partial \psi}{\partial \xi^\nu} = A_\nu, \quad \nu = 2, ... \tag{10}$$

where ξ^ν are the coordinates in the direction normal to the trajectory, A_ν are the quantities that depend on the specific features of the system and external force actions.

Eqs. (9), (10) can be convoluted into the relation

$$d\psi = A_\mu \, d\xi^\mu, \quad (\mu = 1, \nu) \tag{11}$$

where $d\psi$ is the differential expression $d\psi = (\partial\psi/\partial\xi^\mu)d\xi^\mu$.

Relation (11) can be written as

$$d\psi = \omega \tag{12}$$

Here $\omega = A_\mu \, d\xi^\mu$ is the differential form of the first degree.

Since the balance conservation laws are evolutionary ones, the relation obtained is also an evolutionary relation.

Relation (12) was obtained from the equation of the balance conservation laws for energy and linear momentum. In this relation the form ω is that of the first degree. If the equations of the balance conservation laws for angular momentum be added to the equations for energy and linear momentum, this form in the evolutionary relation will be the form of the second degree. And in combination with the equation of the balance conservation law of mass this form will be the form of degree 3.

Thus, in the general case the evolutionary relation can be written as

$$d\psi = \omega^p \tag{13}$$

where the form degree p takes the values $p = 0, 1, 2, 3$. (The evolutionary relation for $p = 0$ is similar to that in the differential forms, and it was obtained from the interaction of energy and time.)

In relation (12) the form ψ is the form of zero degree. And in relation (13) the form ψ is the form of $(p - 1)$ degree.

Let us show that *the evolutionary relation obtained from the equation of the balance conservation laws turns out to be nonidentical.*

To do so we shall analyze relation (12).

Let us consider the commutator of the form $\omega = A_\mu d\xi^\mu$. The components of the commutator of such a form (as it was pointed above) can be written as follows:

$$K_{\alpha\beta} = \left(\frac{\partial A_\beta}{\partial \xi^\alpha} - \frac{\partial A_\alpha}{\partial \xi^\beta} \right) \tag{14}$$

(here the term connected with the nondifferentiability of the manifold has not yet been taken into account).

The coefficients A_μ of the form ω have been obtained either from the equation of the balance conservation law for energy or from that for linear momentum. This means that in the first case the coefficients depend on the energetic action and in the second case they depend on the force action. In actual processes energetic and force actions have different nature and appear to be inconsistent. The commutator of the form ω constructed of the derivatives of such coefficients is nonzero. This means that the differential of the form ω is nonzero as well. Thus, the form ω proves to be unclosed.

This means that the evolutionary relation cannot be an identical one. In the left-hand side of this relation it stands a differential, whereas in the right-hand side it stands an unclosed form, which is not a differential.

A role of the evolutionary relation obtained consists in the following. It enables one to describe the evolutionary processes in material medium, which lead to emergence of physical structures that form physical fields. From the nonidentical evolutionary relation the identical relations, which contain closed exterior forms, are obtained (under the degenerate transformations). The emergence of the closed exterior form points to a rise to the physical structure.

Here it should be noted that the evolutionary nonidentical relation is a selfvarying relation. This plays a governing role while describing the evolutionary processes.

Selfvariation of the evolutionary nonidentical relation

The evolutionary nonidentical relation is selfvarying, because, firstly, it is nonidentical, namely, it contains two objects one of which appears to be unmeasurable, and, secondly, it is an evolutionary relation, namely, a variation of any object of the relation in some process leads to variation of another object and, in turn, a variation of the latter leads to variation of the former. Since one of the objects is an unmeasurable quantity, the other cannot be compared with the first one, and hence, the process of mutual variation cannot terminate.

Varying the evolutionary form coefficients leads to varying the first term of the evolutionary form commutator (see (7),(14)). In accordance with this variation it varies the second term, that is, the metric form of manifold varies. Since the metric form commutators specifies the manifold differential characteristics, which are connected with the manifold deformation (as it has been pointed out, the commutator of the zero degree metric form specifies the bend, that of second degree specifies various types of rotation, that of the third degree specifies the curvature), this points to the manifold deformation. This means that it varies the evolutionary form basis. In turn, this leads to variation of the evolutionary form, and the process of intervariation of the evolutionary form and the basis is repeated. Processes of variation of the evolutionary form and the basis are controlled by the evolutionary form commutator and it is realized according to the evolutionary relation.

A significance of the evolutionary relation selfvariation consists in the fact that in such a process it can be realized conditions under which the identical relation is obtained from the nonidentical relation. These are conditions of degenerate transformation.

Obtaining an identical relation from a nonidentical one

To obtain the identical relation from the evolutionary nonidentical relation, it is necessary that a closed exterior differential form would be derived from the evolutionary differential form, which is included into evolutionary relation. However, as it has been shown above, the evolutionary form cannot be a closed

form. For this reason the transition from the evolutionary form is possible only to an *inexact* closed exterior form, which is defined on pseudostructure.

To the pseudostructure it is assigned a closed dual form (whose differential vanishes). For this reason the transition from the evolutionary form to a closed inexact exterior form proceeds only when the conditions of vanishing the dual form differential are realized, in other words, when the metric form differential or commutator becomes equal to zero.

Since the evolutionary form differential is nonzero, whereas the closed exterior form differential is zero, a transition from the evolutionary form to the closed exterior form is allowed only under *degenerate transformation*. The conditions of vanishing the dual form differential (the additional condition) are the conditions of degenerate transformation.

Such conditions can just be realized under selfvariation of the nonidentical evolutionary relation.

As it has been already mentioned, the evolutionary differential form ω^p, involved into nonidentical relation (13), is an unclosed one. The commutator, and hence the differential, of this form is nonzero. That is,

$$d\omega^p \neq 0 \tag{15}$$

If the conditions of degenerate transformation are realized, then from the unclosed evolutionary form one can obtain a differential form closed on pseudostructure. The differential of this form equals zero. That is, it is realized the transition

$d\omega^p \neq 0 \rightarrow$ (degenerate transformation) $\rightarrow d_\pi \omega^p = 0$, $d_\pi{}^* \omega^p = 0$

On the pseudostructure π evolutionary relation (13) transforms into the relation

$$d_\pi \psi = \omega_\pi^p \tag{16}$$

which proves to be an identical relation. Indeed, since the form ω_π^p is a closed one, on the pseudostructure this form turns out to be a differential of some differential form. In other words, this form can be written as $\omega_\pi^p = d_\pi \theta$. Relation (16) is now written as

$$d_\pi \psi = d_\pi \theta$$

There are differentials in the left-hand and right-hand sides of this relation. This means that the relation is an identical one.

From evolutionary relation (13) it is obtained the identical on the pseudostructure relation. Here it should be noted that in this case the evolutionary relation itself remains to be nonidentical one. (At this point it should be emphasized that differential, which equals zero, is an interior one. The evolutionary form commutator becomes zero only on the pseudostructure. The total evolutionary form commutator is nonzero. That is, under degenerate transformation the evolutionary form differential vanishes only *on pseudostructure*. The total differential of the evolutionary form is nonzero. The evolutionary form remains to be unclosed.)

The transition from nonidentical relation (13) to identical relation (16) means the following. Firstly, it is from such a relation that one can obtain the differential $d_\pi \psi$ and find the desired function ψ_π (a potential). And, secondly, an emergence of the closed (on pseudostructure) inexact exterior form ω_π^p (right-hand side of relation (16)) points to an origination of the conservative object. This object is a conservative quantity (the closed exterior form ω_π^p) on the pseudostructure (the dual form $^*\omega^p$, which defines the pseudostructure). This object is an example of the physical structure.

As conditions of degenerate transformation (additional conditions) it can serve any symmetries of the evolutionary form coefficients or its commutator. While describing material system such additional conditions are related, for example, to degrees of freedom of the material system.

Mathematically to the conditions of degenerate transformation it is assigned the requirement that some functional expressions become equal to zero. Such functional expressions are Jacobians, determinants, the Poisson brackets, residues, and others.

The degenerate transformation is realized as a transition to nonequivalent coordinate system: a transition from the accompanying noninertial coordinate system to the locally inertial that. Evolutionary relation (13) and condition (15) relate to the system being tied to the accompanying manifold, whereas identical relations (16) can relate only to the locally inertial coordinate system being tied to pseudostructure.

Thus, while selfvariation of the evolutionary nonidentical relation the dual form commutator can vanish (due to the symmetries of the evolutionary form coefficients or its commutator). This means that it is formed the pseudostructure on which the differential form turns out to be closed. The emergence of the form being closed on pseudostructure points out to origination of the physical structures.

Causality of origination of physical structures and of forming physical fields

Since closed inexact exterior forms corresponding to physical structure are obtained from the evolutionary relation for the material system, it follows that physical structures are generated by the material systems. (This is controlled by the conservation laws.)

The mechanism of this process involves the following steps.

1) The external actions onto the material system are transformed into the unmeasurable quantity that acts as an internal force and brings the material system into the nonequilibrium state. (A nonzero value of the evolutionary form commutator. Nonidentical evolutionary relation obtained from balance conservation laws).

2) Selfvariation of the nonequilibrium state of the material system. The deformation of accompanying manifold. (Selvariation of the nonidentical evolu-

tionary relation. The topological properties of the evolutionary form commutator.)

3) Realization of the degrees of freedom of the material system in the process of selvariation of the nonequilibrium state of the system itself. (Degenerate transformations).

4) Transition of the material system from the nonequilibrium state into the locally equilibrium one: transition of an internal force into a potential force. The emergence of the physical structures. (Formation of the closed (on pseudostructure) inexact exterior form and obtaining the state differential, which specifies a state of material system).

In material system the origination of a physical structure reveals as a new measurable and observable formation that spontaneously arises in the material system. {*As the examples it can be fluctuations, pulsations, waves, vortices, creating massless particles.*}.

In the physical process this formation is spontaneously extracted from the local domain of the material system and so it allows the local domain of material system to get rid of an internal force and come into a locally equilibrium state. The formation created in a local domain of the material system (at the cost of an unmeasurable quantity that acts in the local domain as an internal force) and liberated from that, begins acting onto the neighboring local domain as a force. This is a potential force, this fact is indicated by the double meaning of the closed exterior form (on one hand, a conservative quantity, and, on other hand, a potential force). (This action was produced by the material system in itself, and therefore this is a potential action rather than an arbitrary one). The transition of the material system from nonequilibrium into a locally equilibrium state (which is indicated by the formation of a closed form) means that the unmeasurable quantity described by the nonzero commutator of the nonintegrable differential form ω^p, that acts as an internal force, transforms into the measurable quantity. It is evident that this is just the measurable quantity, which acts as a potential force. In other words, the internal force transforms into a potential force. The neighboring domain of the material system works over this action, which appears to be external with respect to that. If in the process the conditions of conjugacy of the balance conservation laws turn out to be satisfied again, the neighboring domain will create a formation by its own, and this formation will be extracted from this domain. In such a way the formation can move relative to the material system. (Waves are the example of such motions).

The physical structures are generated by numerous local domains of the material system and at numerous instants of realizing various degrees of freedom of the material system. It is evident that they can generate fields. In this manner physical fields are formed. To obtain the physical structures that form a given physical field, one has to analyze the material system corresponding to this field and the appropriate evolutionary relation. In particular, to obtain the thermodynamic structures (fluctuations, phase transitions, etc), one has to analyze the evolutionary relation for the thermodynamic systems, to obtain the

gas dynamic ones (waves, jumps, vortices, pulsations) one has to employ the evolutionary relation for gas dynamic systems, for the electromagnetic field one must employ a relation obtained from equations for charged particles.

Characteristics of physical structure

Since the closed exterior differential form, which corresponds to the physical structure arisen, was obtained from the nonidentical relation that involves the evolutionary form, it is evident that the physical structure characteristics must be connected with those of the evolutionary form and of the manifold on which this form is defined, with the conditions of degenerate transformation and with the values of commutators of the evolutionary form and the manifold metric form.

The conditions of degenerate transformation, as it was said before, determine the pseudostructures. The first term of the evolutionary form commutator determines the value of the discrete change (the quantum), which the quantity conserved on the pseudostructure undergoes when transition from one pseudostructure to another. The second term of the evolutionary form commutator specifies the characteristics that fixes the character of the initial manifold deformation, which took place before the physical structure arose. (Spin is such an example).

A discrete (quantum) change of a quantity proceeds in the direction that is normal (more exactly, transverse) to the pseudostructure. Jumps of the derivatives normal to the potential surfaces are examples of such changes.

Classification of physical structures

Closed forms that correspond to physical structures are generated by the evolutionary relation having the parameter p, which defines a number of interacting balance conservation laws. Therefore, the physical structures can be classified by the parameter p.

The other parameter is a degree of closed forms generated by the evolutionary relation.

To determine this parameter one has to consider the problem of integration of the nonidentical evolutionary relation.

Under degenerate transformation from the nonidentical evolutionary relation one obtains a relation being identical on pseudostructure. Since the right-hand side of such a relation can be expressed in terms of differential (as well as the left-hand side), one obtains a relation that can be integrated, and as a result he obtains a relation with the differential forms of less by one degree. The relation obtained after integration proves to be nonidentical as well. The resulting nonidentical relation of degree $(p-1)$ (relation that includes the forms of the degree $(p-1)$) can be integrated once again if the corresponding degenerate transformation has been realized and the identical relation has been formed.

By sequential integrating the evolutionary relation of degree p (in the case of realization of the corresponding degenerate transformations and forming the identical relation), one can get closed (on the pseudostructure) exterior forms of degree k, where k ranges from p to 0.

Thus, one can see that physical structures, to which there are assigned the closed (on the pseudostructure) exterior forms, can depend on two parameters. These parameters are the degree of evolutionary form p (in the evolutionary relation) and the degree of created closed forms k.

In addition to these parameters, another parameter appears, namely, the dimension of space. If the evolutionary relation generates the closed forms of degrees $k = p$, $k = p - 1$, ..., $k = 0$, to them there are assigned the pseudostructures of dimensions $(N - k)$, where N is the space dimension. {It is known that to the closed exterior differential forms of degree k there are assigned skew-symmetric tensors of rank k and to corresponding dual forms there do the pseudotensors of rank $(N - k)$, where N is the space dimensionality. The pseudostructures correspond to such tensors, but only on the space formed.}

Forming of pseudometrical manifolds and metric spaces and physical fields

As mentioned before, the additional conditions, namely, the conditions of degenerate transformation, specify the pseudostructure. But at every stage of the evolutionary process it is realized only one element of pseudostructure, namely, a certain minipseudostructure.

While varying the evolutionary variable the minipseudostructures form the pseudostructure. The example of minipseudoctructure is the wave front. The wave front is an eikonal surface (the level surface), i.e. the surface with a conservative quantity.

Manifolds with closed metric forms are formed by pseudostructures. They are obtained from manifolds with unclosed metric forms. In this case the initial (accompanying) manifold (on which the evolutionary form is defined) and the manifold with closed metric forms originated (on which the closed exterior form is defined) are different spatial objects.

It takes place a transition from the initial (accompanying) manifold with unclosed metric form to the pseudostructure, namely, to the manifold with closed metric forms being created. Mathematically this transition (degenerate transformation) proceeds as *a transition from one frame of reference to another, nonequivalent, frame of reference.*

The pseudostructures, on which the closed *inexact* forms are defined, form the pseudomanifolds. (Integral surfaces, pseudo-Riemann and pseudo-Euclidean spaces are the examples of such manifolds). In this process the dimensions of the manifolds formed are connected with the evolutionary form degree.

To the transition from pseudomanifolds to metric space it is assigned a transition from closed *inexact* differential forms to *exact* exterior differential forms.

It was shown above that the evolutionary relation of degree p can generate (in the presence of the degenerate transformations) closed forms of the degree $0, ..., p$. While generating closed forms of sequential degrees $k = p, k = p - 1, ..., k = 0$ the pseudostructures of dimensions $(n + 1 - k)$ are obtained. As a result of transition to the exact closed form of zero degree the metric structure of the dimension $n + 1$ is obtained. Under the influence of an external action (and with the availability of degrees of freedom) the material system can transfer the initial inertial space into the space of the dimension $n + 1$.

Sections of the cotangent bundles (Yang-Mills fields), cohomologies by de Rham, singular cohomologies, pseudo-Riemannian and pseudo-Euclidean spaces, and others are examples of the psedustructures and spaces that are formed by pseudostructures. Euclidean and Riemannian spaces are examples of metric manifolds that are obtained when going to the exact forms. Here it should to be noted that the examples of pseudometric spaces are potential surfaces (surfaces of a simple layer, a double layer and so on). In these cases the type of potential surfaces is connected with the above listed parameters.

What can be said about the pseudo-Riemannian manifold and Riemannian space?

The distinctive property of the Riemannian manifold is an availability of the curvature. This means that the metric form commutator of the third degree is nonzero. Hence, it does not equal zero the evolutionary form commutator of the third degree, which involves into itself the metric form commutator. That is, the evolutionary form, which enters into the evolutionary relation, is unclosed, and the relation is nonidentical. When realizing pseugostructures of the dimensions $1, 2, 3, 4$ and obtaining the closed inexact forms of the degrees $k = 3, k = 2, k = 3, k = 4$ the pseudo-Riemannian space is formed, and the transition to the exact form of zero degree corresponds to the transition to the Riemannian space.

It is well known that while obtaining the Einstein equations it was suggested that there are fulfilled the conditions: The Bianchi identity is satisfied, the coefficients of connectedness are symmetric, the condition that the coefficients of connectedness are the Christoffel symbols, and an existence of the transformation, under which the coefficients of connectedness vanish. These conditions are the conditions of realization of the degenerate transformations for nonidentical relations obtained from the evolutionary relation of the degree $p = 3$ and after going to the exact relations. In this case to the Einstein equation it is assigned the identical relation of the first degree.

As it has been pointed above, from the physical structures, to which the closed forms are assigned and which are generated by material systems, the physical fields are formed. Physical fields form the unified whole with corresponding manifolds. Since the closed metric form is dual with respect to some closed exterior differential form, the metric forms cannot become closed by themselves, independent of the closed exterior differential form. This proves that manifolds with closed metric forms are connected with the closed exterior differential forms. This indicates that the fields of conservative quantities, such

as physical fields, are formed from closed exterior forms at the same instant of time when the manifolds are created from the pseudoctructures. (The specific feature of the manifolds with closed metric forms that have been formed is that they can carry some information.)

References

1. Petrova L. I., Exterior and evolutionary skew-symmetric differential forms and their role in mathematical physics.
http://arXiv.org/pdf/math-ph/0310050
2. Petrova L. I., Invariant and evolutionary properties of the skew-symmetric differential forms.
http://arXiv.org/pdf/math.GM/0401039
3. Cartan E., Les Systemes Differentials Exterieus ef Leurs Application Geometriques. -Paris, Hermann, 1945.
4. Bott R., Tu L. W., Differential Forms in Algebraic Topology. Springer, NY, 1982.
5. Schutz B. F., Geometrical Methods of Mathematical Physics. Cambridge University Press, Cambridge, 1982.
6. Encyclopedia of Mathematics. -Moscow, Sov. Encyc., 1979 (in Russian).
7. Wheeler J. A., Neutrino, Gravitation and Geometry. Bologna, 1960.
8. Petrova L. I., Identical and nonidentical relations. Nondegenerate and degenerate transformations. (Properties of the skew-symmetric differential forms).
http://arXiv.org/pdf/math.GM/0404109
9. Petrova L. I., Conservation laws. Their role in evolutionary processes. (The method of skew-symmetric differential forms).
http://arXiv.org/pdf/math-ph/0311008
10. Fock V. A., Theory of space, time, and gravitation. -Moscow, Tech. Theor. Lit., 1955 (in Russian).
11. Clark J. F., Machesney M., The Dynamics of Real Gases. Butterworths, London, 1964.
12. Dafermos C. M. In "Nonlinear waves". Cornell University Press, Ithaca-London, 1974.
13. Tolman R. C., Relativity, Thermodynamics, and Cosmology. Clarendon Press, Oxford, UK, 1969.